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RADIATION COUPLED VISCOUS FLOWS*

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Abstract—A general formulation and method for solving viscous, radiating, non-similar boundary-layer flows is developed and applied to both the stagnation region and a flat plate. The formulation and solutions provide for an emitting and absorbing gas including the entire range of optical thicknesses, from thin to thick. Results indicate the direct contribution of radiation to the net heat flux as well as the decrease of temperatures, their gradients and the related conductive heat transfer to the wall.

NOMENCLATURE

Bo, Boltzmann number, $\dfrac{\rho_e u_e h_0}{\sigma T_0^4}$;

Bu, boundary-layer Bouguer number, $\tau_L/\sqrt{(Re)}$;

f, dimensionless stream function, equation (7);

g, dimensionless enthalpy, equation (8);

h, enthalpy;

j, geometric parameter; (0, 1) for (two-dimensional, axisymmetric) flow;

k, coefficient of thermal conductivity;

P, static pressure;

Pr, Prandtl number, $\dfrac{\mu C_p}{k}$;

$\mathbf{q}$, heat flux vector;

r, distance from axis of symmetry;

R, body nose radius;

$\bar{R}$, universal gas constant;

Re, Reynolds number,
$$(Re)_1 = \frac{\rho_e U_e R}{\mu_e}, (Re)_2 = \frac{\rho_e u_e x}{\mu_e};$$

T, temperature;

u, v, velocity components parallel and normal to body surface;

U_e, reference velocity, $U_e^2 = 2T_0\bar{R}$;

x, y, distance along and normal to body surface.

Greek symbols

Γ, radiation–convection interaction parameter, τ_L/Bo;

η, transformed coordinate normal to body surface, equation (6);

κ, Planck's mean volumetric absorption coefficient;

Λ, ratio of (density viscosity) products, equation (9);

μ, coefficient of viscosity;

ξ, transformed coordinate along flow direction, equation (5);

ρ, density;

σ, Stefan–Boltzmann constant;

τ, optical thickness, equation (16);

τ_e, optical thickness of the boundary layer;

τ_L, characteristic optical thickness, $(\tau_L)_1 = R\kappa_0, (\tau_L)_2 = x\kappa_0$;

ψ, stream function.

Subscripts and superscripts

$'$, derivative with respect to η;

c, conduction;

D, diffusion approximation;

e, outer edge of boundary layer;

R, radiation;

w, wall;

0, stagnation reference state; also transparent layer approximation;

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1, stagnation region;
2, flat plate;
I, II, first and second order thin gas approximation;
∞, free stream property.

INTRODUCTION

A RADIATION coupled flow is one in which the flowing substance is sufficiently hot to both emit and absorb radiant energy. Such flows are encountered in a number of natural and engineering sciences including planetary and stellar atmospheres, oceanography, radiant boilers, furnaces and the flow of molten glass and plastics. Recent interest has grown steadily with the foreseen speed increase of re-entry space vehicles. Large superorbital velocities imply that the temperature of the gas surrounding the vehicle is sufficiently high to excite even a relatively weak radiator such as air to emit and absorb. Earlier investigations [1, 2] have shown that the flow field effects are governed to a large extent by a simple radiation–convection interaction parameter, Γ, with radiation of considerable importance whenever $\Gamma \geqslant 0{\cdot}1$. On this basis Fig. 1 indicates the regions of atmospheric flight for which radiation coupling will be of interest, based on conditions behind a normal shock where the radiant intensities are largest. Of course, Γ will be smaller in magnitude for less severely disturbed flows.

It is characteristic of the viscous boundary layer that temperatures of the order of the stagnation temperature may be present, thus introducing direct radiation heating of the boundary as well as alteration of the layer structure and so conductive heating. A reduction of the thermal gradients is expected due to radiation exchanges and the resulting relative magnitude of radiative and conductive energy fluxes is of major interest.

Several solutions of the radiating boundary layer over a flat plate have appeared recently in the literature, each using different techniques, see [3–5]. They are concerned with flow conditions somewhat different from those assumed here thus prohibiting quantitative comparisons. For example, Pai and Tsao consider very high temperature external flows, $5000 \leqslant T_e \leqslant$ °K 40000, while Cess assumes negligible viscous dissipation. Our flow model for a flat plate will come closest perhaps to Oliver and McFadden except for their low $[(\gamma^{-1}/2)M^2 = 0{\cdot}01]$ and intermediate $[(\gamma^{-1}/2)M^2 = 3{\cdot}0]$ external flow velocities in contrast to the very high velocity condition $[(u_e^2/h_0) \to 2]$ to be encountered in re-entry trajectories. The present purpose is to devise a unified analysis and method of solution which will be applicable to a boundary-layer flow with a pressure gradient, and allow for a range of absorption (or optical thickness), without depending on highly complex and/or time consuming numerical schemes. The analysis will be restricted, however, to thin viscous and thermal boundary layers for which the Navier–Stokes equations can be simplified using familiar boundary-layer approximations and for which, it will be shown, the use of a one-dimensional radiation model is justified. For simplicity the gas is assumed to be "gray" and in thermodynamic equilibrium.

GOVERNING EQUATIONS

The general conservation equations for a single component radiating gas are available in [1]. For two-dimensional or axisymmetric, steady boundary-layer flow the descriptive conservation relations are:

$$\frac{\partial}{\partial x}(\rho u r^j) + \frac{\partial}{\partial y}(\rho v r^j) = 0 \tag{1}$$

$$\rho u\left(\frac{\partial u}{\partial x}\right) + \rho v\left(\frac{\partial u}{\partial y}\right) = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right), \tag{2}$$

$$\rho u\left(\frac{\partial h}{\partial x}\right) + \rho v\left(\frac{\partial h}{\partial y}\right) = u\frac{\partial p}{\partial x} + \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + \mu\left(\frac{\partial u}{\partial y}\right)^2 - \nabla\cdot\mathbf{q}_R = 0. \tag{3}$$

A reduction of the number of independent variables is obtained upon introducing a stream

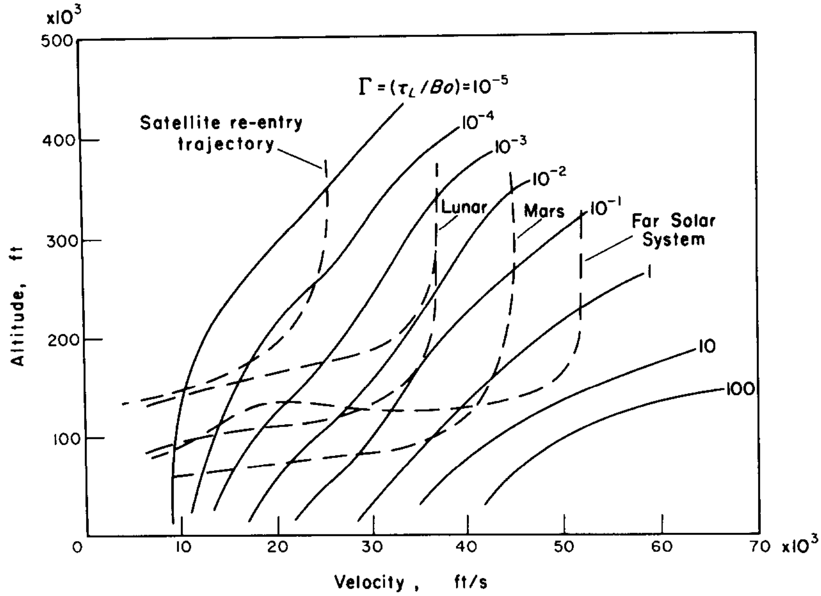

FIG. 1. Γ, based on properties behind a normal shock and $R = 1$ ft.

function ψ on the basis of the continuity equation

$$\frac{\partial\psi}{\partial y} = \rho u r^j; \qquad \frac{\partial\psi}{\partial x} = -\rho v r^j. \tag{4}$$

Despite the expectation of a non-similar formulation a similarity transformation may be used to reduce equations (1–3) to a more amenable form for numerical evaluations. For this purpose we introduce the Lees–Dorotnitsyn transformation [6] to a new system (ξ, η), through

$$\xi \equiv \int_0^x \rho_e u_e \mu_e r^{2j}\, dx, \tag{5}$$

$$\eta \equiv \frac{\rho_e u_e r^j}{\sqrt{(2\xi)}} \int_0^y \frac{\rho}{\rho_e}\, dy. \tag{6}$$

We define also a modified stream function, f, by

$$f(\xi, \eta) \equiv \frac{\psi}{\sqrt{(2\xi)}} \tag{7}$$

and use the following convenient normalizations:

$$g(\xi, \eta) \equiv \frac{h}{h_0}, \tag{8}$$

$$\Lambda \equiv \frac{\rho\mu}{\rho_e\mu_e} \tag{9}$$

$$\overline{\nabla \cdot \mathbf{q}_R} \equiv \frac{\nabla \cdot \mathbf{q}_R}{\sigma\kappa_0 T_0^4} \tag{10}$$

where h_0, the reference enthalpy, is taken as the stagnation enthalpy of the free stream.

Equations (2) and (3) now become

$$\frac{\partial}{\partial\eta}\left(\Lambda\frac{\partial^2 f}{\partial\eta^2}\right) + f\left(\frac{\partial^2 f}{\partial\eta^2}\right) + \left(\frac{2\xi}{u_e}\frac{\partial u_e}{\partial\xi}\right) \times \left[\frac{\rho_e}{\rho} - \left(\frac{\partial f}{\partial\eta}\right)^2\right] + 2\xi\left(\frac{\partial f}{\partial\xi}\frac{\partial^2 f}{\partial\eta^2} - \frac{\partial f}{\partial\eta}\frac{\partial^2 f}{\partial\xi\partial\eta}\right) = 0 \tag{11}$$

$$\frac{\partial}{\partial\eta}\left(\frac{\Lambda}{Pr}\frac{\partial g}{\partial\eta}\right)+f\left(\frac{\partial g}{\partial\eta}\right)+\frac{u_e^2}{h_0}\left[\Lambda\left(\frac{\partial^2 f}{\partial\eta^2}\right)^2 - \left(\frac{2\xi}{u_e}\frac{\partial u_e}{\partial\xi}\right)\left(\frac{\rho_e}{\rho}\frac{\partial f}{\partial\eta}\right)\right]+2\xi\left(\frac{\partial f}{\partial\xi}\frac{\partial g}{\partial\eta} - \frac{\partial f}{\partial\eta}\frac{\partial g}{\partial\xi}\right)-\left(\frac{2\xi\kappa_0}{\rho_e u_e \mu_e r^{2j}}\right)\left(\frac{\sigma T_0^4}{\rho_e u_e h_0}\right) \times \left(\frac{\rho_e}{\rho}\right)\nabla\cdot\mathbf{q}_R = 0. \quad (12)$$

$\nabla\cdot\mathbf{q}_R$ as derived from the basic radiative transfer equations, is presented in [1].

The associated boundary conditions are

$$\left.\begin{aligned} &\text{at } \eta = 0: \; f = \text{const.}, \quad g = g_w, \quad \frac{\partial f}{\partial\eta} = 0 \\ &\text{at } \eta \to 1: \qquad\qquad\quad g = g_e, \quad \frac{\partial f}{\partial\eta} = 1. \end{aligned}\right\} \quad (13)$$

RADIATIVE ENERGY FLUX

In a boundary layer the Prandtl reduction of the Navier–Stokes equations implies viscous and thermal layers which are thin and for which the dominant variations occur across the layer. It follows that changes of the radiative heat flux in the layer are such that

$$\frac{\partial(q_R)_x}{\partial x} \ll \frac{\partial(q_R)_y}{\partial y}. \quad (14)$$

We shall further assume that the flow external to the viscous layer and the wall adjacent to the layer are non-reflective and at a constant temperature. Under such conditions $\nabla\cdot\mathbf{q}_R$ reduces to (see for example [2, 7])

$$\nabla\cdot\mathbf{q}_R(\tau) \simeq \frac{\partial(\mathbf{q}_R)_y}{\partial y} = \sigma\kappa\left\{4T^4(\tau) - 2T_w^4 E_2(\tau) - 2T_e^4 E_2(\tau_e-\tau) - 2\int_0^{\tau_e} T^4(t)\,E_1(|\tau-t|)\,dt\right\}. \quad (15)$$

τ, the "optical thickness", is defined as

$$\int_0^y \kappa(y)\,dy$$

and in terms of the transformed coordinates (ξ, η) is given by

$$\tau = \frac{\kappa_0\sqrt{(2\xi)}}{\rho_e u_e r^j}\int_0^\eta\left(\frac{\kappa}{\kappa_0}\,\frac{\rho_e}{\rho}\right)dy \quad (16)$$

E_n is the exponential integral [8]

$$E_n(Z) = \int_0^1 \mu^{(n-2)}\,e^{-Z/\mu}\,d\mu. \quad (17)$$

Equations (11, 12, 15) represent a formulation of the radiation coupled viscous flow problem but unfortunately involve partial integro-differential equations the straight-forward numerical solution of which requires a large expense of computation time (e.g. see [9]). Appreciable simplifications result upon approximating $\nabla\cdot\mathbf{q}_R(y)$ by localized terms involving only properties at the point y (or τ). The governing equations without the integral term reduce to a set of partial differential equations and the computation times become comparable to those for classical boundary layers.

Localized expressions for the radiation absorption integral are readily obtained for the limits of optically thin ($\tau_e \ll 1$) or optically thick ($\tau_e \gg 1$) layers. These may be summarized as:

1. Transparent layer approximation ($\tau_e \ll 1$) [2]

$$(\nabla\cdot\mathbf{q}_R)_0 = 4\sigma\kappa T^4. \quad (18)$$

2. First order "thin" layer approximation ($\tau_e^2 \ll 1$) [7]

$$(\nabla\cdot\mathbf{q}_R)_\text{I} = \sigma\kappa[4T^4 - 2T_w^4 - 2T_e^4]. \quad (19)$$

3. Second order "thin" layer approximation ($\tau_e^3 \ll 1$) [7]

$$(\nabla\cdot\mathbf{q}_R)_\text{II} = \sigma\kappa\Big[4T^4 - 2T_w^4 - 2T_e^4 + \tfrac{1}{2}(\tau_e-\tau)^2\{\tfrac{3}{2} - \gamma - \log(\tau_e-\tau)\} \times \frac{dT^4}{d\tau}(\tau_e) - \tfrac{1}{2}\tau^2\{\tfrac{3}{2} - \gamma - \log\tau\} \times \frac{dT^4}{d\tau}(0)\Big] \quad (20)$$

where γ is Euler's constant (0·577216 . . .).

4. "Thick" layer (Rosseland or diffusion) approximation ($\tau_e \gg 1$) [2]

$$(\nabla \cdot \mathbf{q}_R)_D = -\frac{4}{3}\nabla\left(\frac{\sigma}{\kappa}\nabla T^4\right). \qquad (21)$$

5. Infinitely thick layer ($\tau_e \to \infty$)

$$\lim_{\tau_e \to \infty} (\nabla \cdot \mathbf{q}_R) = 0. \qquad (22)$$

The first and second order "thin" layer approximations follow upon twice integrating by parts the integral term of equation (15) and expanding E_2 and E_3 for small values of the argument (as, e.g. [8] p. 255).

A more generalized approximation, applicable for the entire range of optical thicknesses, is based on matching an initially unknown function, T^4, by a polynomial, $f(\tau)$, using Lagrange's interpolation formula

$$T^4 \simeq f(\tau) = \sum_{i=0}^{n} A_i \tau^i. \qquad (23)$$

The coefficients A_i are determined by making $f(\tau)$ take on the same values as T^4 for $n+1$ predetermined distinct values of $\tau(\tau = \tau_0, \tau_1, \ldots \tau_n)$, resulting in $n+1$ linear algebraic equations for the A_i. One may start by taking the uncoupled similarity solution for the temperature distribution in combination with τ for the same layer.

Introducing equation (23) into the integral term of equation (15) and integrating by parts, results in the nth term:

$$\int_0^{\tau_e} t^n E_1(|\tau - t|)\, dt =$$

$$= n!\left\{\sum_{k=0}^{n} \frac{1}{k!}\{\tau^k E_{n+2-k}(0) \times [(-1)^{n-k} + 1] - \tau_e^k \times E_{n+2-k}(\tau_e - \tau)\} - E_{n+2}(\tau)\right\}. \qquad (24)$$

This reduces the system to a set of partial differential equations for which existing numerical techniques are applicable, proceeding by iteration or successive approximations. Each iteration implies a new distribution of T^4, and therefore new A_i for the next iteration. The procedure is repeated until self consistency is obtained; namely, until two successive solutions are virtually identical. Accuracy increases, of course, with increasing terms, n, in equation (23), at the cost of additional computation time.

SOLUTIONS FOR STAGNATION REGION AND FLAT PLATE BOUNDARY LAYERS

Equations (11) and (12) simplify considerably for both the stagnation region and flat plate boundary layers.

Stagnation region

Assume a Newtonian velocity distribution of the form $u_e = U_e\, x/R$; restricting the analysis to the vicinity of the stagnation point where T_e, ρ_e and μ_e are nearly constant and $r \simeq x$, the radiation interaction parameter becomes

$$\left(\frac{2\xi\kappa_0}{\rho_e u_e \mu_e r^{2j}}\right)\left(\frac{\sigma T_0^4}{\rho_e u_e h_0}\right) = \frac{1}{j+1}(\kappa_0 R)\left(\frac{\sigma T_0^4}{\rho_e U_e h_0}\right) = \frac{1}{j+1}\frac{(\tau_L)_1}{B_0} \equiv \frac{1}{j+1}\Gamma_1 \qquad (25)$$

and the optical thickness, equation (16), is now given by

$$\tau_1 = \frac{(\tau_L)_1}{\sqrt{[2(Re)_1]}}\int_0^{\eta}\left(\frac{\kappa}{\kappa_0} \times \frac{\rho_e}{\rho}\right) d\eta. \qquad (26)$$

Γ_1 is clearly ξ (or x) independent and so is $\overline{\nabla \cdot \mathbf{q}_R}$ when it is based on a one-dimensional radiation model. Similarity solutions can be sought therefore for the stagnation region radiating boundary layer. For an axisymmetric body ($j = 1$) and with $(u_e^2/h_0) \ll 1$ (resulting, e.g. from a strong shock wave which preceeds the body) the governing equations reduce finally to

$$(Af'')' + ff'' + \tfrac{1}{2}(g - f'^2) = 0 \qquad (27)$$

$$\left(\frac{A}{Pr}g'\right)' + fg' - \frac{1}{2}\Gamma_1 \frac{g}{g_e}\, \overline{\nabla \cdot \mathbf{q}_R} = 0. \qquad (28)$$

The boundary conditions are still given by relations (13) with $g_e = 1{\cdot}0$. $g_w = 0{\cdot}1$ was used for all stagnation region solutions.

Flat plate

With a low temperature external flow, ρ_e, u_e and μ_e maintained constant and $j = 0$ the radiation interaction parameter is given by

$$\left(\frac{2\xi\kappa_0}{\rho_e u_e \mu_e}\right)\left(\frac{\sigma T_0^4}{\rho_e u_e h_0}\right) = (2\kappa_0 x)\left(\frac{\sigma T_0^4}{\rho_e u_e h_0}\right) = 2\frac{(\tau_L)_2}{Bo} \equiv 2\Gamma_2 \qquad (29)$$

and the optical thickness

$$\tau_2 = \frac{(\sqrt{2})(\tau_L)_2}{[\sqrt{(Re)_2}]}\int_0^\eta \left(\frac{\kappa}{\kappa_0}\frac{\rho_e}{\rho}\right)\mathrm{d}\eta. \qquad (30)$$

Γ_2 is specifically x (or ξ) dependent such that the energy equation remains in partial differential form.

With $(\partial u_e/\partial\xi) = 0$ and $(u_e^2/h_0) \to 2$ for a high speed external flow the governing equations are

$$f''' + ff'' = 0 \qquad (31)$$

$$g'' + fg' + 2(f'')^2 - 2\xi f'\frac{\partial g}{\partial \xi} - 2\Gamma_2\frac{g}{g_e}\overline{\nabla \cdot \mathbf{q}_R} = 0 \qquad (32)$$

subject to boundary conditions (13). $g_e = 0{\cdot}02$ was used for all flat plate solutions.

For the numerical solution of equation (32) changes relative to ξ may be treated as finite differences while retaining differentials in the η direction. The partial differential equation is then approximated by a total differential equation, as first introduced by Hartree [10] and later used for a variety of boundary-layer problems by Clutter and Smith [11, 12].

Note that the parameter $Bu = [\tau_L/\sqrt{(Re)}]$ determines for both cases the magnitude of the layer's optical thickness, equations (26) and (30), and may be considered as an effective reference optical thickness, known also as a boundary-layer Bouguer number [5]. $Bu < 0$ (0·01) and

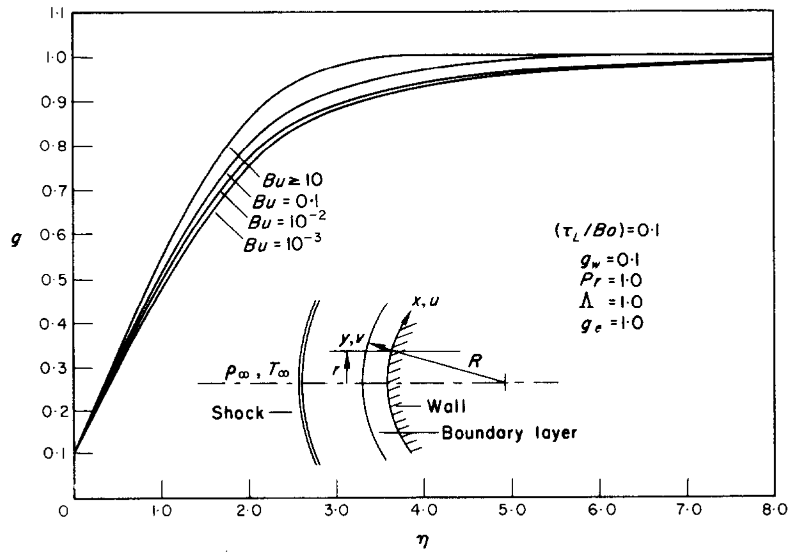

FIG. 2. Stagnation region enthalpy profiles and coordinate system, $\tau_L/Bo = 0{\cdot}1$.

$Bu > 0$ (10) implies usually optically thin and thick layers respectively.

For both flow regimes the solutions are restricted to constant Λ and Pr parameters (e.g. $\Lambda = Pr = 1$) corresponding to a single component non-reacting gas.

$\overline{\nabla \cdot \mathbf{q}_R}$ was computed at each point by means of equation (15) in which the integral terms was replaced by approximation (24) with $n = 2$. The equations were solved numerically. Computation times were comparable with those required for the solution of non-radiating boundary-layer equations: 5–10 s for a complete run on an IBM 7094.

The results obtained are presented in Figs. 2–6. These show the increasing importance of radiation with increasing interaction parameter Γ, but with modifications according to the

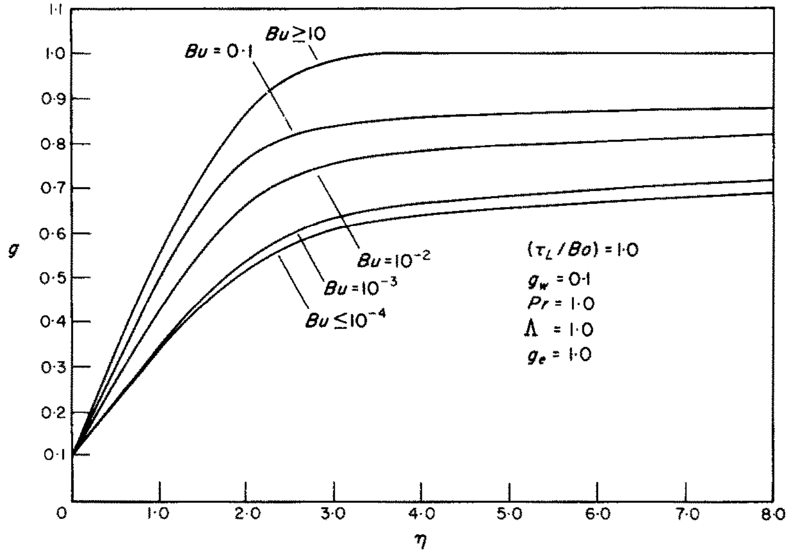

FIG. 3. Stagnation region enthalpy profiles, $\tau_L/Bo = 1{\cdot}0$.

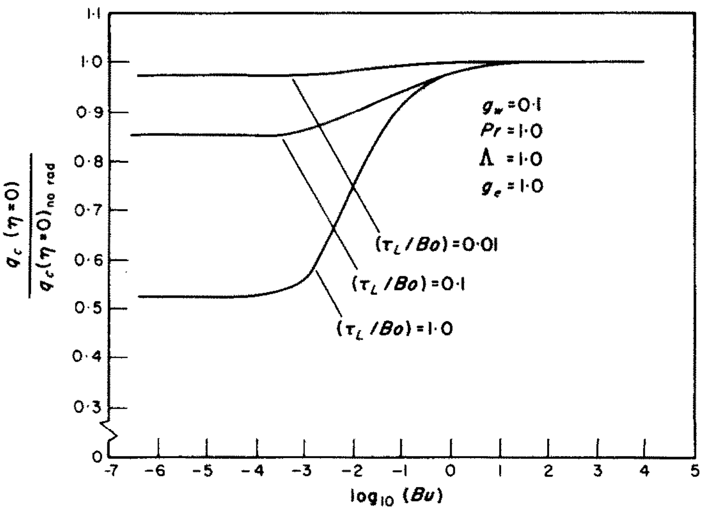

FIG. 4. Effect of radiation on conductive heat transfer to the wall (stagnation region).

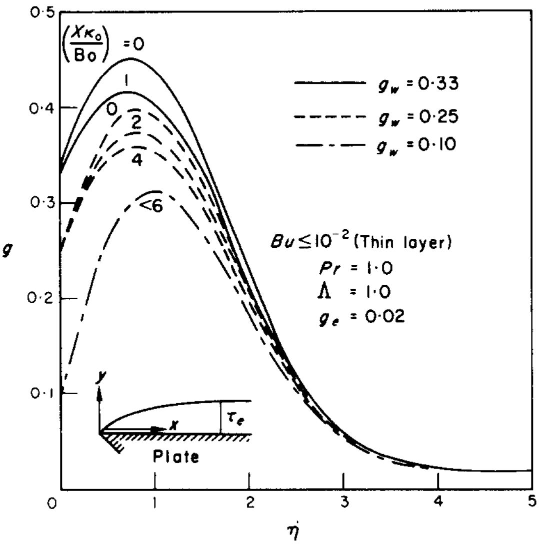

FIG. 5. Flat plate enthalpy profiles.

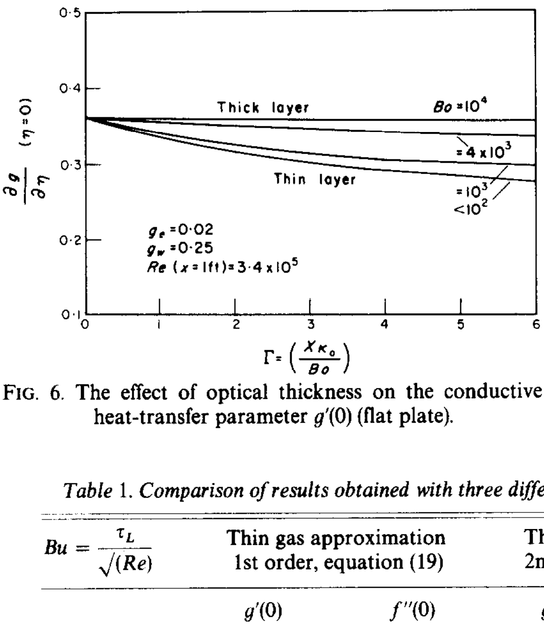

FIG. 6. The effect of optical thickness on the conductive heat-transfer parameter $g'(0)$ (flat plate).

optical thickness parameter *Bu*. The presence of radiation in the layer has a tendency to reduce the enthalpies (or temperatures), their slopes, and increase the thickness of the thermal boundary layer, as compared to the non-radiating case (Figs. 2, 3 and 5). (Figure 3 shows only the inner portion of the profiles; particularly for the optically thin layers, g reaches the asymptotic value of unity at considerably higher η. The figure was limited to "small" η values to accentuate the differences in the enthalpy slopes near the wall.) From the changes in the enthalpy slopes near the wall the effect of radiation on the conductive heat transfer to the wall can be deduced. The coupling between radiation transfer and convection in the flow field generally reduces the conductive heat transfer to the walls (Figs. 4, 6). For a given interaction parameter Γ, the effect of radiation is largest for small *Bu*, since there is then little reabsorption. The radiant energy flux in the layer is at its peak. With increasing *Bu*, but maintaining the same Γ level, the overall loss of energy through emission decreases. Finally, for very large *Bu*, the radiation energy is locally "trapped" and the net effect of radiation decreases sharply, consistent with equation (22).

In Table 1 the results of three approximations are compared; the generalized approximation with $n = 2$ agrees closely with the thin layer

Table 1. Comparison of results obtained with three different approximations. (Stagnation region with $\tau_L/Bo = 0.1$)

$Bu = \frac{\tau_L}{\sqrt{(Re)}}$	Thin gas approximation 1st order, equation (19)		Thin gas approximation 2nd order, equation (20)		Generalized approximation ($n = 2$)	
	$g'(0)$	$f''(0)$	$g'(0)$	$f''(0)$	$g'(0)$	$f''(0)$
10^{-5}	0·3829	0·5803	0·3829	0·5803	0·3829	0·5803
10^{-4}	0·3834	0·5806	0·3833	0·5805	0·3833	0·5805
10^{-3}	0·3876	0·5829	0·3874	0·5823	0·3872	0·5826
10^{-2}	0·4060	0·5931	0·4057	0·5930	0·4055	0·5930
10^{-1}	0·4205	0·6009	0·4201	0·6003	0·4197	0·6002
1	0·4204	0·6009	0·4247	0·6043	0·4412	0·6132
10			0·4303	0·6071	0·4494	0·6183
10^2			0·4307	0·6073	0·4495	0·6184

For non-radiating layer—$g'(0) = 0.4495, f''(0) = 0.6184$

approximations up to $Bu = O$ (0·1) and approaches the correct limit of zero radiative transfer as $Bu \gg 1$.

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Résumé—On expose une formulation générale et une méthode pour résoudre les écoulements de couche limite visqueuse, en non-similitude avec rayonnement et on l'applique à la fois à la région d'arrêt et à une plaque plane. La formulation et les solutions sont fournies pour un gaz émetteur et absorbant en incluant la gamme totale d'épaisseurs optiques. Les résultats indiquent la contribution directe du rayonnement au flux de chaleur net aussi bien que la décroissance des températures, leurs gradients et le transport de chaleur par conduction vers la paroi.

Zusammenfassung—An allgemeinen Formulierungs- und Lösungsmethoden wurden für zähe, nicht-ähnliche Grenzschichtströmungen mit Strahlung gefunden und sowohl auf den Staubereich als auch auf die ebene Platte angewendet. Die Formulierung und die Lösungen berücksichtigen emittierendes und absorbierendes Gas im gesamten Bereich der optischen Dicken von dünn bis dick. Die Ergebnisse zeigen das direkte Mitwirken der Strahlung am Wärmefluss sowie die Abnahme der Temperaturen, ihrer Gradienten und des damit verbundenen Wärmeübergangs an die Wand.

Аннотация—Предложены общая формулировка и метод решения задачи о вязких неавтомодельных течениях излучающего пограничного слоя. На этой основе проведено аналитическое исследивание гидродинамики и теплообмена в области передней критической точки на плоской пластине. Результаты имеют широкую применимость в случаях излучающего и поглощающего газа во всем диапазоне значений оптической толщины, от больших до малых. Установлена доля теплового излучения в общем энергетическом потоке. Подсчитаны также снижение температуры, её градиентов и кондуктивная составляющая теплопереноса к стенке.